

MAR THOMA RESIDENTIAL SCHOOL, TIRUVALLA

FIRST TERMINAL EXAMINATION 2017-18

CLASS: XI

MATHEMATICS

TIME:3h

MAXMARKS:80

SECTION-A

1. i) Solve for x : $2^{2x+3} + 2^{x+3} = 1 + 2^x$

ii) Prove using mathematical induction that $x^n - y^n$ is divisible by $x - y$ for $n \in N$.

iii) Find the sum of the series $2 + 7 + 12 + \dots + 102$.

iv) If the 3rd term of a GP is square of the 1st term . If the 5th term is 64.

Find the series?

v) If the roots of the equation $ax^2 + bx + c = 0$ be in the ratio 2:3. Show that $6b^2 = 25ac$.

(2 × 5)

2. i) The product of 1st 3 terms of a GP is 1000. If we add 6 to its 2nd term and 7 to its 3rd term the resulting three terms form an AP series. Find the GP?

ii) Using mathematical induction prove that

$$1.4.7 + 2.5.8 + 3.6.9 + \dots + n(n+3)(n+6) = \frac{n}{4}(n+1)(n+6)(n+7) \text{ for all } n \in N$$

(2 × 3)

3. i) If α and β are the roots of the equation $3x^2 - 4x + 1 = 0$. Form an equation whose roots are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$.

ii) $\frac{1}{x+y}, \frac{1}{2y}, \frac{1}{y+z}$ are three consecutive terms of an AP . prove that x, y, z are

three consecutive terms of a GP?

(2 × 3)

4. i) Using the principle of mathematical induction that $n(n+1)(2n+1)$ is divisible by 6 for all $n \in N$.

ii) The roots α and β of the quadratic equation $2x^2 - 5x + k = 0$ are connected by

the relation $2\alpha + \beta = 1$. Show that $k = -12$

(2 × 3)

5. i) Find the sum of the series whose n^{th} term is $T_n = 4n^3 + 6n^2 + 2n$
 ii) Find three integers in AP such that their sum is 27 and product is 504? (2 × 3)
6. i) If one root of $x^2 + (5a + 2)x + (5a + 2) = 0$ is 5 times the other. Find 'a'?.
 ii) In a GP, the 3rd term exceeds the 2nd by 6 and 2nd term exceeds the first by 9. Find the first term, common ratio and S_{10} ? (2 × 3)

SECTION-B

1. Express $\frac{2\pi}{15}$ in degree measure. (2)
2. Write the power set of $\{8, 3, 1\}$.
3. In a group of 400 people, 250 can speak Hindi and 200 can speak English. Everyone can speak atleast one language. How many people can speak both Hindi and English?
4. A pendulum 36 cm long oscillates through an angle of 10 degrees. Find the length of the path described by its extremity.
5. Let $A = \{1, 2, 3, 4, 6\}$. Let R be the relation on A defined by $\{(a, b) : a, b \in A, a \text{ divides } b\}$ (i) Write in the roster form (ii) Find the domain of R (iii) Find the range.
6. Prove that $(1 + \cot A + \tan A)(\sin A - \cos A) = \frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A}$
7. If $5\sin\theta = 3$, find the value of $\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta}$
8. Prove that $\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A$
9. Find the value of θ lying between 0° and 720° when $\sin\theta = \frac{1}{\sqrt{2}}$
10. A sector of a circle of radius r m is bounded by an arc AB and by two radii OA and OB, at an angle β radian. Given that the perimeter of the sector is 18m and that the area of sector is 8m^2 . calculate the value of β and r
11. If A, B, C, D are the angles of a cyclic quadrilateral., show that $\cos A + \cos B + \cos C + \cos D = 0$
12. Let $A = \{a, b, c\}$, $B = \{c, d\}$ and $C = \{d, e, f\}$. Find i $(A \times B) \cap (A \times C)$ (ii) $(A \times C) \cap (A \times B)$ (3 * 10)
13. Evaluate $\frac{\cos 300^\circ \cot 300^\circ \tan(-420^\circ)}{\sin 135^\circ \sec 225^\circ \operatorname{cosec} 240^\circ}$ (4 * 2)