MAR THOMA RESIDENTIAL SCHOOL, TIRUVALLA

FIRST MODEL EXAMINATION 2017-18

MATHEMATICS

(Three Hours)

81

CLASS XII **MARKS: 100**

SECTION A

(Answer all Questions)

Question 1 (10 × 2)

a) Let $*$ be a binary operation on N defined by $a * b = H.C.F$ of a and b. Is $*$ commutative? Does there exist identity for this binary operation?

b) Construct a 3 × 4 matrix A whose elements are given by $a_{ij} = i - j$.

c) Solve $\cos^{-1}(\sin\cos^{-1}x) = \frac{\pi}{6}$

d) If the matrix $\begin{bmatrix} 6 & x & 2 \\ 2 & -1 & 2 \\ 10 & 5 & 2 \end{bmatrix}$ is singular, find x.

e) Evaluate $\int_0^{\frac{\pi}{2}} cos^2 x dx$

f) Find $\frac{dy}{dx}$ if $y = (sinx)^{(sinx)^{(sinx)^{(sinx)\ldots\infty}}}$

g) Find the sum of the order and degree of the differential equation

$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0$

h) Find the intervals in which the function $f(x) = x^2 + 2x - 5$ is increasing.

i) Find P(B/A) if (i) A is a subset of B (ii) A and B are mutually exclusive.

j) A pair of dice is thrown. What is the probability of getting an even number on the first die or a total of 8?

Question 2 (4)

If $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$, show that $fof(x) = x$, for all $x \neq \frac{2}{3}$. What is the inverse of f?

Question 3 (4)

a) Using properties of determinants prove that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

OR

b) Show that $\begin{vmatrix} x^2 & y^2 & z^2 \\ yz & zx & xy \\ x & y & z \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$

(4)

Question 4

If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$ prove that $x^2 - y^2 - z^2 + 2yz\sqrt{1-x^2} = 0$

(4)

Question 5

a) Verify Rolle`s Theorem for the function $f(x) = e^x cosx, x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

OR

b) Prove that the function $f(x) = |x-1|, x \in R$ is continuous at $x = 1$ but not differentiable.

(4)

Question 6

If $y = x^3 \log\frac{1}{x}$ prove that $x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3x^2 = 0$

(4)

Question 7

a) Evaluate $\int \frac{x^2+1}{(x-1)^2(x+3)} dx$.

OR

b) Evaluate $\int_0^2 (x^2+3)dx$ as the limit of a sum.

(4)

Question 8

a) Find the equations of the normals to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$.

OR

b) A circular disc of radius 3 cm is heated. Due to expansion its radius increases at the rate of 0.05cm/s. Find the rate at which its area increases when the radius is 3.2cm.

Question 9 (4)

Solve the differential equation $x^2dy + (xy + y^2)dx = 0$ when $x = 1, y = 1$.

Question 10 (4)

An article manufactured by a company consists of two parts A and B. In the process of manufacture of part A, 9 out of 104 parts may be defective. Similarly 5 out of 100 are likely to be defective in the manufacture of part B. Calculate the probability that the article manufactured will not be defective.

Question 11 (6)

a) Find A^{-1} and hence solve the equation $4x + 2y + 3z = 2, x + y + z = 1,$ $3x + y - 2z = 5$.

$$A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -2 \end{bmatrix}$$

OR

b) Using elementary row operations find the inverse of $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

(6)

Question 12

a) Show that the altitude of a right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.

OR

b) An open topped box is to be made by removing equal squares from each corner of a 3m by 8m rectangle sheet of aluminium and by folding up the sides. Find the volume of the largest such box.

(6)

Question 13

Evaluate $\int \frac{\cos x-\cos 2x}{1-\cos x} dx$.

Question 14 (6)

The probability that a bulb produced by a factory will fuse in 100 day of use is 0.05. Find the probability that out of such 5 bulbs after 100 days of use (i) none fuse (ii) not more than one fuse (iii) more than one fuse (iv) atleast one fuse.

SECTION B

Question 15 (6)

a) Find a unit vector perpendicular to the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ where $\vec{a}=3i+2j+2k$, $\vec{b}=i+2j-2k$.

b) If the cartesian equation of a line is $3x+1=6y-2=1-z$ then reduce it to vector form.

c) Find the equation of a plane which passes through the point $(3,2,0)$ and contains the line $\frac{x-3}{1}=\frac{y-6}{5}=\frac{z-4}{4}$.

Question 16 (4)

a) Show that $(\vec{a}\times\vec{b})^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} \end{vmatrix}$

OR

b) Prove that $[\vec{a}-\vec{b},\vec{b}-\vec{c},\vec{c}-\vec{a}]=0$

Question 17 (4)

a) If the equation of a plane is $-2x+6y-3z=-7$ find (i) length of the perpendicular drawn from the origin to the plane (ii) direction cosines of the vector normal to the plane (iii) co-ordinates of the foot of the perpendicular drawn from the origin to the plane (iv) equation of the plane in normal form.

OR

b) Find the distance of the point $(3,4,4)$ from the point where the line joining the points $A(3,-4,-5)$ and $B(2,-3,1)$ intersects the plane $2x+y+z=7$.

Question 18 (6)

Find the smaller area of the region bounded by the curves $x=4y-y^2$ and the y-axis.

SECTION-C

Question 19 (6)

a) Given that the total cost function for x units of a commodity is $C(x)=\frac{x^3}{3}+3x^2-7x+16$. Find (i) Marginal cost (ii) Average cost.

b) Two regression lines are represented by $2x+3y-10=0$ and $4x+y-5=0$. Find the correlation coefficient between X and Y.

c) The total variable cost of manufacturing x units in a firm is Rs.$3x+\frac{x^5}{25}$. Show that Average Variable cost increases with output x.

Question 20 (4)

a) Find the two regression for the the following observations by method of least squares. (1,3),(2,7),(3,9),(4,10),(5,11).

OR

b) If the regression lines of a bivariate distribution are $4x-5y+33=0$ and $20x-9y-107=0$ then find (i) $\bar{X}$ and $\bar{Y}$ (ii) value of x when y=7. (iii)varience of y when $\sigma x=3$.

Question 21 (4)

a) A manufacturer can sell 'x' items per day at a price 'p' rupeew each where $p=125-\frac{5}{3}x$. The cost of production for 'x' items is $500+13x+0.2x^2$. (i) Find how much he should produce to have a maximum profit, assuming all items produced are sold. (ii) What is the maximum profit?

OR

b) The fixed cost of a new product is Rs. 30,000 and variable cost per unit is Rs. 800. If the demand function is $p(x)=4500-100x$, find break even values.

Question 22 (6)

A firm deals with two kinds of fruit juices- pineapple and the orange juice. These are mixed and two types of mixtures are obtained, which are sold as soft drinksA and B. one tin of A needs 4 litres of pineapple juice and 1 litre of orange juice. One tin of B needs 2 litres os pineapple and 3 litres of orange juice. The firm has only 46 litres of pineapple juice and 24m litres of orange juice. Each tin of A and is sold at a profit of Rs.4 and Rs.3 respectively. How many tins of A and B should the firm produce to maximize profit? Formuklate the linear programming problem and solve it graphically using iso-cost or iso- profit approach.