MARTHOMA RESIDENTIAL SCHOOL, TIRUVALLA

FIRST TERMINAL EXAMINATION 2017-18

MATHEMATICS

Standard: XII Science

Time: 3 *hrs*
Marks: 80

SECTION A

(Answer all Questions)

(i) Matrix R(t) is given by $R(t)=\begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix}$. Show that $R(s).R(t) = R(s+t)$.

(ii) Find x if $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & x \end{bmatrix}$ is a singular matrix.

(iii) If $A=\begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$, $B=[1 \quad 3 \quad -6]$ find $B'A'$.

(iv) Evaluate $\int sin^3 x\, dx$.

(v) Evaluate $\int \frac{sin\theta}{\sqrt{4cos^2\theta-1}} d\theta$. $(5 \times 2 = 10)$

(i) Find the adjoint of the matrix $A=\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ (3)

(ii) If $A=\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$. Show that f (A) = 0 and hence find A^{-1}. (3)

(i) Using properties of determinants prove that

$$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$$

OR

(ii) $$\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix} = (b^2-ac)(ax^2+2bxy+cy^2).$$ (6)

(i) If (a,b), (a',b') and $(a-a', b-b')$ are collinear show that $ab' = a'b$. (2)

(ii) Prove using properties of determinants that

$$\begin{vmatrix} 1+sin^2x & cos^2x & 4sin2x \\ sin^2x & 1+cos^2x & 4sin2x \\ sin^2x & cos^2x & 1+4sin2x \end{vmatrix} = 2+4\sin 2x$$ (2)

(iii) Evaluate $\int \frac{4x \tan^{-1} x^2}{1+x^4} dx$ (2)

(i) Evaluate $\int \frac{3x-2}{(x+3)(x+1)^2} dx$ (3)

(ii) Evaluate $\int e^{2x} cos^2 x dx$. (3)

(i) If $A = \begin{bmatrix} 4 & -5 & -11 \\ 1 & -3 & 1 \\ 2 & 3 & -7 \end{bmatrix}$, find A^{-1} and hence solve $4x - 5y - 11z = 12$, $x - 3y + z = 1, 2x + 3y - 7z = 2$. (4)

(ii) Evaluate $\int \frac{1}{sinx+tanx} dx$. (2)

Section B
(Answer all Questions)

. Prove that $\tan^{-1}\left[\frac{x}{\sqrt{a^2-x^2}}\right] = \sin^{-1}\frac{x}{a}$.

. Show that the function f(x) defined by $f(x) = \begin{cases} \frac{sinx}{x} + cosx, & x > 0 \\ 2, & x = 0 \\ \frac{4(1-\sqrt{1-x})}{x}, & x < 0 \end{cases}$ is continuous at x = 0.

. A die is thrown twice and sum of the numbers appearing is observed to be 7. What is the conditional probability that the number 2 has appeared atleast once?

. Find the magnitude of <BAC where A, B, C are the vertices of ΔABC having position vectors as the points (2, 1, 3), (4, 1, 3) and (0, 2, 1) respectively.

. The sum of mean and variance of a binomial distribution of 18 trials is 10. Find the distribution.

(5 × 2 = 10)

. Prove that $2\tan^{-1}\left(\frac{1}{3}\right) + \cot^{-1} 4 = \tan^{-1}\left(\frac{16}{13}\right)$. (3)

. The odds against a husband who is 45 years old, living till he is 70 are 7:5 and the odds against his wife who is now 36 living till she is 61 are 5:3. Find the probability that (i) the couple will be alive 25 yrs hence. (ii) at least one of them will be alive 25 yrs hence.

OR

The probability of A, B, C solving a problem are $\frac{1}{3}, \frac{2}{7}$ and $\frac{3}{8}$ respectively. If all the three try to solve the problem simultaneously, find the probability that exactly one of them will solve it.

. Prove that $\vec{a}.(\vec{b}+\vec{c}) \times (\vec{a}+2\vec{b}+3\vec{c}) = [\vec{a}, \vec{b}, \vec{c}]$. (3)

(3)

). a) Show that the points $A(-1,4,-3), B(3,2,-5), C(-3,8,-5) and\ D(-3,2,1)$ are co-planar.

b) Find a unit vector parallel to the xy plane and perpendicular to the vector $4\hat{\imath} - 3\hat{\jmath} + \hat{k}$.

OR

a) Find a unit vector coplanar with $\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and perpendicular to $\hat{\imath} + \hat{\jmath} + \hat{k}$.

a) If $\vec{a}$ and $\vec{b}$ are unit vectors inclined at an angle θ then prove that $\cos\frac{\theta}{2} = \frac{1}{2}|\vec{a} + \vec{b}|$. (6)

0. a) The chances of X, Y, Z becoming managers of certain company are 4:2:3. The probability that bonus scheme will be introduced if X, Y, Z become managers are 0.3, 0.5 and 0.8 respectively. If the bonus scheme has been introduced what is the probability that X is appointed as manager? (3)

b) A student is given a true-false examination with 8 questions. If he gets 6 or more correct answers he passes the exam. Given that he guesses correct answers. Find the probability that the student failed in the examination. (3)

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