MARTHOMA RESIDENTIAL SCHOOL- TIRUVALLA
SECOND MODEL EXAMINATION 2018-19

CLASS:12 **MATHEMATICS** **TIME :3H**

MAXMARK:100

The question paper consists of 3 sections A,B,C. The candidates are required to attempt all the questions from Section A and all the questions from either from Section B or Section C

SECTION-A

Question -1

i. Using determinants find 'a' so that the points (1,5) (2,4) and (a,2) are collinear.

ii. If f: x→2x, g: x→ x^2 and h: x→x+1.Find ho(gof) and (hog)of. What do you notice.

iii. Using L'Hospital's rule evaluate $\lim_{x\to 0}(1+sinx)^{cotx}$.

iv. Solve : $\cos^{-1}[sin\cos^{-1}x]=\frac{\pi}{6}$.

v. Verify that the function y = x sinx is a solution of the differential equation $x\,y^1 = x\sqrt{x^2-y^2}+y$.

vi. Using proper substitution differentiate $\sin^{-1}(\frac{2^{x+1}}{1+4^x})$ with respect to 'x'.

vii. Evaluate. $\int 6^{x+\tan^{-1}x}\,(\frac{x^2+2}{x^2+1})\,dx$.

viii. Find the absolute maximum and minimum values of the function $f(x)=(x-1)^2+3$ in the interval [-3,1].

ix. Evaluate P(E∪ F), if 2P(E) =P(F) =$\frac{5}{13}$ and P(E/F)= $\frac{2}{5}$.

x. Find the rate of change of volume of a sphere with respect to its diameter.

(2× 10)

Question -2

Let ∗ be defined on the set A= {1,2,3,4,5} by a∗ b = HCF(a,b).

1. Is ∗ a binary operation.
2. Is ∗ commutative.
3. Find (2 ∗ 3) ∗ 4 and 2∗ (3∗ 4).
4. Find (2∗ 3)∗ (4∗ 5) .

(4)

Question -3

Use lagrange's mean value theorem to determine a point 'p' on the curve y= $\sqrt{x-2}$ defined in th interval [2,3] where the tangent is parallel to the chord joining the endpoints on the curve?

(4

Question -4

a) Prove that the function f(x) =$|x-1|$,x∈ R is continuous at x = 1 but not differentiable

OR

b) Find 'a' and 'b' if the function given by $f(x) = \begin{cases} ax^2 + b & if\ x < 1 \\ 2x + 1 & if\ x \geq 1 \end{cases}$ is differentiable at $x = 1$?

(4)

Question-5

a) Using properties of determinants prove that

$$\begin{vmatrix} 1 + \sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1 + \cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1 + 4\sin 2x \end{vmatrix} = 2 + 4\sin 2x$$

(4)

OR

b) Using properties of determinants prove that

$$\begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix} = (1 - a^3)^2$$

Question-6

If $\log y = \tan^{-1} x$ prove that $(1+x^2)\frac{dy^2}{dx^2} + (2x-1)\frac{dy}{dx} = 0$. (4)

Question – 7

Solve the differential equation $(3xy + y^2)dx + (x^2 + xy)dy = 0$. (4)

Question-8

Find the equation of tangents to the curve $y = x^3 + 2x - 4$ which are perpendicular to the line $x + 14y + 3 = 0$. (4)

Question-9

In a college 70% students pass in physics, 75% pass in mathematics, and 10% students fail in both. One student is chosen at random. What is the probability that

i) He passes in physics and mathematics.

ii) He passes in mathematics given that he passes in physics.

iii) He passes in physics given that he passes in mathematics. (4)

Question-10

a. Evaluate $\int cos\sqrt{x}\, dx$.

OR

b. Evaluate $\int \frac{2x+5}{x^2+7x+12} dx$. (4)

Question-11

a. Solve given system of equations by using matrix method $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1,$

$\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$, ,x, y, z≠ 0

OR

b. If A= $\begin{bmatrix}1 & 2 & -3\\ 2 & 3 & 2\\ 3 & -3 & -4\end{bmatrix}$ find A-1 and hence solve the following system of linear equations

x+ 2y-3z =-4, 3x-3y-4z =11, 2x+3y+2z = 2. (6)

Question-12

a. Using property evaluate $\int_0^{\frac{\pi}{2}} \frac{xsinxcosx}{\sin^4 x+\cos^4 x}dx$.

OR

b. Evaluate $\int_0^{\frac{\pi}{2}} \frac{1}{sinx+cosx}dx$. (6

Question-13

Show that the semi vertical angle of a cone of maximum volume and given slant height is given by $\tan^{-1}\sqrt{x}$. (6)

Question-14

Five bad eggs are mixed with 10 good ones. If three eggs are drawn one by one with replacement. Find the probability distribution of the number of good eggs drawn. (6)

SECTION-B

Question-15

a. If $\vec{a}=\hat{\imath}+\hat{\jmath}+2\hat{k}$ and $\vec{b}=2\hat{\imath}+\hat{\jmath}-2\hat{k}$ find a unit vector in the direction of $2\vec{a}-\vec{b}$.

b. Write the value of $\hat{\imath}\cdot(\hat{\jmath}\times\hat{k})+\hat{\jmath}\cdot(\hat{k}\times\hat{\imath})+\hat{k}\cdot(\hat{\jmath}\times\hat{\imath})$.

c. Find a vector of magnitude 6 which is perpendicular to both the vectors $2\hat{\imath}-\hat{\jmath}+2\hat{k}$ and $4\hat{\imath}-\hat{\jmath}+3\hat{k}$. (6)

Question -16

a. For any three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ prove $[\vec{a}-\vec{b},\ \vec{b}-\vec{c},\vec{c}-\vec{a}]\ =0$.

OR

b. Find the volume of the parallelepiped whose edges are represented by the vectors $\vec{a}=2\ \hat{\imath}-3\hat{\jmath}-4\hat{k}$, $\vec{b}=\hat{\imath}+2\hat{\jmath}-\hat{k}$, $\vec{c}=3\hat{\imath}+\hat{\jmath}+2\hat{k}$ (4

Question-17

a. Find the equation of plane through the point (1,2,3) and perpendicular to the planes x+y+2z=3 and 3x+2y+z =4.

OR

b. Find the equation of plane which contains the line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-3}{4}$ and is perpendicular to the plane x+2y+z=12. (4)

Quiestion-18

Sketch the graphs of the curves $y^2=x$ and $y^2=4-3x$ and find the area enclosed between them. (6)

SECTION-C

Question-19

a. $C(x)= 5x+350$ and $R(x)=50x-x^2$ are respectively the total cost and total revenue functions, then find break even values.

b. The total revenue received from the sale of x units of a product is given by $R(x) =20x-0.5x^2$ find
   i) Average revenue
   ii) The marginal revenue
   iii) The actual revenue from selling 15^{th} item.

c. The two lines of regression for a distribution (x,y) are $3x+2y=7$ and $x+4y=9$. Find b_{yx} and b_{xy}?. (6)

Question -20

a. The average cost function AC for a commodity is given by $AC = x+5+\frac{36}{x}$ in terms of output'x'. Find the i) total cost 'C' and MC as function of x. ii) output for which AC increases.

OR

b. Given the total function for 'x'units of a commodity as $C(x)=\frac{1}{3}x^3+3x^2-7x+16$ find i) the marginal cost ii)show that the marginal average cost is given by $\frac{xMC-C(x)}{x^2}$. (4)

Question-21

a. The lines of regression of set of data are $8x-10y+66=0$, $40x-18y=214$. The variance of x is 9. Find i) mean value of x and y ii) co-efficient of b_{xy} and b_{yx}. iii) standard deviation of y . iv) the value of y for x=2

OR

b. For the data given below find regression equation of X on Y. Using the equation calculate the value of X when Y = 15.

X	20	25	30	35	40	45
y	12	14	16	20	22	25

(4)

Question-22

Solve the following problem by ISO-COST/ISO-PROFIT method. Maximize $Z= 50x+15y$ subject to $5x+y\leq 100$, $x+y\leq 60$ $x,y\geq 0$. (6)