MAR THOMA RESIDENTIAL SCHOOL TIRUVALLA

FIRST MODEL EXAMINATION 2019-20

MATHEMATICS

CLASS : 12

(MAX MARK:100)

(TIME ALLOWED: Three hours)

(Candidates are required to attempt all questions from **Section A** and all questions **EITHER** from **Section B OR Section C**)

SECTION – A

Question-1

i. Find the value of (x-y) from the matrix equation
$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} -3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

ii. Solve : $\cos\left[\sin^{-1}\frac{2}{5} + \cos^{-1}x\right] = 0.$

iii. Find x if the points (2,-5) (-4,5) and (x,15) are collinear.

iv. If $*$ is defined on the set R of all real numbers by $a*b = \sqrt{a^2+b^2}$, find the identity element, if it exists in R with respect to $*$.

v. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ and $x \neq y$. Prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$.

vi. Evaluate $\lim_{x\to 0} x\log x$.

vii. Evaluate $\int \frac{1}{\sqrt{3x+4}-\sqrt{3x+1}}dx$.

viii. If P(A) = 0.6 , P(B) = 0.5 and $P(B|A)$= 0.4, find P(A∪B) and $P(A|B)$

ix. The volume of a cube is increasing at a rate $7cm^3$/sec. How fast is the surface area increasing when the length of the edge is 12cm?

x. A single letter is selected at random from the word probability. Find the probability that it is a vowel?

Question-2 [4]

Show that the function f in A=R-$\{\frac{2}{3}\}$ defined as f(x)= $\frac{4x+3}{6x-4}$ is one-one and onto. Hence find f^{-1}?

Question-3 [4]

Prove that $\begin{vmatrix} x-y-z & 2x & 2x \\ 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \end{vmatrix} = (x+y+z)^3.$

Question-4

[4]

If $\tan^{-1}x - \cot^{-1}x = \tan^{-1}\frac{1}{\sqrt{3}}$, $x > 0$, find the value of x and hence find the value of $\sec^{-1}\frac{2}{x}$

Question-5

[4]

(a) Verify Lagrange's mean value theorem for the function $f(x) = 2\sin x + \sin 2x$ in $[0,\pi]$

OR

(b) Find the value of λ so that the function given below is continuous at $x = -1$,

$$f(x) = \begin{cases} \frac{x^2-2x-3}{x+1}, & x \neq -1 \\ \lambda, & x = -1 \end{cases}.$$

Question-6

[4]

If $y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0.$

Question-7

[4]

Evaluate $\int_0^{\frac{\pi}{4}} \log(1+\tan x)\,dx.$

Question-8

(a) Determine the points on the curve $y = x^3 - 3x^2 - 9x + 7$ at which the tangents are parallel to x axis [4]

OR

(b) Sand is pouring from a pipe at the rate of $12 cm^3/sec$. The falling sands forms a cone on the ground in such a way that the height of the cone is always $1/6^{th}$ of the radius of the base. How fast is the height of the sand cone increasing when height is 4cm?

uestion-9

[4]

(a) Find the order and degree of $\frac{d^2y}{dx^2} = 1 + \sqrt{\frac{dy}{dx}}$.

(b) Form the differential equation for the curve $(2x+a)^2 + y^2 = a^2$, where a is the parameter?

Question-10

(a) Find the probability distribution of X in two throws of two dice, where X represent the number of times a total of 9 appears

OR

(b) Determine the binomial distribution where mean is 9 and standard deviation is $\frac{3}{2}$. Al find the probability of obtaining at most one success.

Question-11

(a) Solve : $6x-9y-20z=-4$

$4x-15y+10z=-1$

$2x-3y-5z=-1$

OR

(b) Find the inverse of the following using elementary transformations $A=\begin{bmatrix}2 & 3 & 1\\ 2 & 4 & 1\\ 3 & 7 & 2\end{bmatrix}$.

Question-12

A rectangle is inscribed in a semicircle of radius ' r' with one of its sides in diameter of se circle. Find the dimensions of the rectangle so that its area is maximum. Find its area?

Question-13

(a) $\int \frac{x}{x^4+x^2+1}\,dx$.

OR

(b) $\int \frac{1}{\sin x+\sin 2x}\,dx$.

Question-14

In answering a question on a multiple choice questions test with four choices in each question, out of which only one is correct, a student either guesses or copies or knows th answer. The probability that he guess a answer is ¼ and the probability he copies is also The probability that the answer is correct, given that he copied it is ¾. Find the probabil that he knows the answer to the question given that he correctly answered it?

SECTION – B

Question-15 [3

(a) Find a unit vector in the direction of $\vec{a} - 2\vec{b} + 3\vec{c}$ if $\vec{a} = \hat{\imath} + \hat{\jmath}$, $\vec{b} = \hat{\jmath} + \hat{k}$, $\vec{c} = \hat{\imath} + \hat{k}$.

(b) Find the angle between the line $\vec{r} = (2\hat{\imath} - \hat{\jmath} + 3\hat{k}) + \lambda(3\hat{\imath} - \hat{\jmath} + 2\hat{k})$ and the plan

$\vec{r} \cdot (\hat{\imath} + \hat{\jmath} + \hat{k}) = 3$.

(c) Find the equation of a plane passing through the point $(\hat{\imath} + \hat{\jmath} + \hat{k})$ and perpendicula the vector $6\hat{\imath} + 4\hat{\jmath} + 3\hat{k}$.

Question-16

(a) Find the equation of the plane passing through a point (4,2,4) perpendicular to th planes $\vec{r} \cdot (2\hat{\imath} + 5\hat{\jmath} + 4\hat{k}) = -1$ and $\vec{r} \cdot (4\hat{\imath} - 12\hat{\jmath} - 3\hat{k}) = -2$.

OR

(b) Find the equation of the line passing through the point (1,-1,1) and perpendicula the lines joining the points (4,3,2) (1,-1.0)and (1,2,-1) (2,1,1).

Question-17

Let $\vec{a}$, $\vec{b}$, and $\vec{c}$ be three vectors such that $|\vec{a}| = 1$ $|\vec{b}| = 2$ $|\vec{c}| = 3$. If the projection of $\vec{b}$ a $\vec{a}$ is equal to the projection of $\vec{c}$ along $\vec{a}$ and $\vec{b}$, $\vec{c}$ are perpendicular to each other, then f $|3\vec{a} - 2\vec{b} + 2\vec{c}|$.

Question-18

Find the area under the curve $y = \sqrt{6x + 4}$ above the x axis from $x = 0$ and $x = 2$.

SECTION – C

Question-19 [3

(a) Given the cost function and the revenue function respectively as $C(x) = x + 40$ an $R(x) = 10x - 0.2x^2$ find the break even point?

(b) Find the regression coefficients b_{yx} and b_{xy}. If $\sum x = 30, \sum y = 42,$ $\sum xy = 199, \sum x^2 = 184, \sum y^2 = 318$ and n=6.

(c) The company is selling a certain product. The demand function of the product is linear. The company can sell 2000 units when the price is ₹4 per unit ,it can sell 3(units. Determine i) demand function ii) total revenue function.

Question-20

[4]

(a) Given that the observations are (9,-4)(10,-3)(11,-1)(13,1)(14,3)(15,5)(16,8). Find the two lines of regression. Estimate the value of y when x =13.5.

OR

(b) Two random variables have regression lines 3x+2y-26=0 , 6x+y-31=0. Calculate

(i) The mean value of x and y

(ii) The correlation coefficient

(iii) Standard deviation of y given variance of x=25.

Question-21

[4]

If the demand function is $P = \sqrt{6 - x}$, find at what level of output x, the total revenue will be maximum?

Question-22

[6]

A toy company manufactures two types of dolls A and B. Market test and available resources have indicated that the combined production level should not exceed 1200 dolls per week and demands for the dolls of type B is at most half of that for dolls of type A . Further, the production level of type A can exceed 3 times the production of dolls of other type by at most 600 units. If the company makes a profit of ₹ 12 and ₹16 per doll respectively on dolls A and B. How many of each type of dolls should be produced weekly, in order to maximise the profit?