MAR THOMA RESIDENTIAL SCHOOL, TIRUALLA

FIRST TERMINAL EXAMINATION 2019-20

CLASS:12

MATHEMATICS

TIME: 3H

ATTEMPT ALL THE QUESTIONS **MAXMARK: 80**

SECTION-A

I

1. Find the values of x and y if $\begin{bmatrix} 3x+y & -y \\ 2y-x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$.
2. Without expanding ,show that $\begin{vmatrix} 9 & 9 & 12 \\ 1 & -3 & -4 \\ 1 & 9 & 12 \end{vmatrix} = 0$.
3. Find the principal values of $\tan\left(\sin^{-1}\frac{\sqrt{2}}{2}\right) - \cot\left(\cos^{-1}\frac{\sqrt{2}}{2}\right)$.
4. Given that f: x→3x-2 and g: x→x^5 for all x∈ R, find g∘f(x) and f(g(x)+3).
5. The binary operation $*$:R×R→R is defined as a$*$b = 2a+b. Find (2$*$3) $*$4. (5×2)

II

1. show that $\sin\left(\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{3}{5}\right) = \frac{3+4\sqrt{3}}{10}$. (4)

   OR

   Show that $(AB)^T = B^T A^T$ where A= $\begin{bmatrix} -1 & 3 & 0 \\ -7 & 2 & 8 \end{bmatrix}$, $B = \begin{bmatrix} -5 & 0 \\ 0 & 3 \\ 1 & -8 \end{bmatrix}$

2. use properties of determinants prove that

$$\begin{vmatrix} -a(b^2+c^2-a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2+a^2-b^2) & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2+b^2-c^2) \end{vmatrix} = abc(a^2+b^2+c^2)^3. \quad (4)$$

III

1. Solve by matrix method $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$; $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$; $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$. (4)
2. Find the adjoint of the matrix A= $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ (3)

IV

1. Let Z be the set of all integers and R be the relation on Z defined as (3)

   R = {(a,b): a, b ∈ Z and a-b is divisible by 5}. Prove that R is an equivalence relation.

2. If f: A→ A *and* A= R-{8/5}, show that the function f(x)= $\frac{8x+3}{5x-8}$ is one one and onto. Hence, find f^{-1}.

V

1. If a, b, c are positive and respectively equal to p^{th}, q^{th}, r^{th} terms of a G.P then show that $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$. (4)

2. Prove that $\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$ (4)

OR

Let A= $\begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$, express as the sum of two matrices such that one is symmetric and the other is skew symmetric

SECTION-B

I

1. If $y = e^{x+e^{x+e^{x+\cdots\cdots\cdots\infty}}}$ prove that $\frac{dy}{dx} = \frac{y}{1-y}$.
2. Evaluate $\lim_{x\to\frac{\pi}{4}}(1-\tan x)\sec 2x$.
3. If $x = a(t+\sin t)$; $y = a(1-\cos t)$ find $\frac{dy}{dx}$.
4. A spherical soap bubble is expanding so that its radius is increasing at the rate of 0.02cm/s. At what rate its surface area increasing when radius is 5cm?
5. Find the approximate value of $(82)^{1/4}$ up to places of decimal using differentiation.

(5× 2)

II

1. Show that the function is continuous at and not differentiable at x=2,

$$f(x)=\begin{cases} 3x-2, 0 \le x \le 1 \\ 2x^2 - x, 1 \le x \le 2 \\ 5x-4, x > 2 \end{cases}$$ (4)

2. If $y = \left(x+\sqrt{x^2+1}\right)^m$ show that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2y = 0$. (4)

1. Verify Rolle's theorem for the function $f(x)=x(x+3)e^{-\frac{x}{2}}$ in [-3,0] (4)
2. Find the maximum and minimum value of the function 2x3-24x+107 on [-3,3]. (3)

1. Find the maximum volume of the cylinder which can br inscribed in a sphere of radius $3\sqrt{3}$cm (leave the answer in terms of π). (4)

2. Find a point on the curve $y = x^3-11x+5$ at which the tangent has the equation $y = x-11$. (4)

Find the interval in which the following function is strictly increasing or strictly decreasing $f(x) = x^3-12x^2+36x+17$, $x \in R$ (4)

Differentiate the function with respect to x , $y = \cot^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}$. (3)

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