MAR THOMA RESIDENTIAL SCHOOL TIRUVALLA

SECOND MODEL EXAMINATION 2019-20

MATHEMATICS

(MAX MARK:100)

CLASS:12 (TIME ALLOWED: Three hours)

(Candidates are required to attempt all questions from **Section A** and all questions **EITHER** from **Section B OR Section C**)

SECTION-A

QUESTION-1

i) Let a $*$b denotes the larger 'a and b' if a o b = a$*$b + 3, then write the value of (5)o(10) where $*$ and o are binary operations.

ii) If the matrix A = $\begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ -b & 1 & 0 \end{bmatrix}$ is skew symmetric find the values of a and b.

iii) Find the value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$.

iv) Find the value of x if $\begin{vmatrix} x & 4 \\ 2 & 2x \end{vmatrix} = 0$.

v) Show that the function $f(x) = \begin{cases} 3x-2 & if\ 0 < x \le 1 \\ 2x^2 - x & if\ 1 < x \le 2 \\ 5x-4 & if\ x > 2 \end{cases}$ is continuous at x = 2.

vi) Use differentials to find the value of $\sqrt{0.037}$.

vii) Form the differential equation representing the family of parabolas $x^2 = 4ay$.

viii) In a class there are 1000 students , out of which 430 are girls. It is known that out of 430, 10% of the girls study in class XII. What is the probability that a student chosen randomly studies in class XII given that the chosen student is a girl?

ix) If A and B are two events such that p(A) = $\frac{1}{3}$, p(B)= $\frac{1}{4}$ and p(A∪B)= $\frac{1}{2}$, then show that A and B are independent events?

x) Show that $f(x) = x^3 - 3x^2 + 3x - 100$ is increasing on R. (2 × 10)

QUESTION-2

Show that the relation R on $\mathfrak{R}$ defined as R= {(a,b): a$\le b$} is reflexive and transitive but not symmetric. (4)

QUESTION-3

If $\tan^{-1} a + \tan^{-1} b + \tan^{-1} c = \pi$. prove that $a + b + c = abc$. (4)

QUESTION-4

Using properties of determinants prove that $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$. (4)

QUESTION-5

If $f(x) = \begin{cases} x^2+3x+a, & x \le 1 \\ bx+2, & x > 1 \end{cases}$ is differentiable everywhere, then find a and b.

OR

It is given that Rolle's theorem holds good for the function f(x)= $x^3 + ax^2 + bx$, $x \in [1,2]$ at the point x = 4/3, find a and b. (4)

QUESTION-6

If $y = \sin(\sin x)$ prove that $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y \cos^2 x = 0$. (4)

QUESTION-7

Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{\tan x}} dx$. (4)

QUESTION-8

Find the equation of the normal to the curve $x^2 + 2y^2 - 4x - 6y + 8 = 0$ at the point where abscissa is 2.

OR

The radius of a circle is increasing uniformily at the rate of 4 cm/sec. Find the rate at which the area of the circle is increasing when radius is 8 cm?

QUESTION-9

Find the particular solution of the differential equation $\frac{dx}{dy} + x \cot y = 2y + y^2 \cot y \ (\neq 0)$ given $x = 0$ when $= \frac{\pi}{2}$. (4)

OR

In a bank, principal increases continuously at the rate 5% per year. In how many years will ₹ 1000 double itself?

QUESTION-10

In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is 5/6. What is the probability that he will knock down fewer than 2 hurdles? (4)

$\vec{r} = 3\hat{\imath} - 2\hat{\jmath} + 4\hat{k} + \lambda(-2\hat{\imath} + \hat{\jmath} + 2\hat{k})$ and $\hat{r} = 3\hat{\imath} + 3\hat{\jmath} - 2\hat{k} + \mu(3\hat{\imath} - 2\hat{\jmath} + 6\hat{k})$

(c) Find the equation of the plane passing through the line of intersection of the planes $x + 2y + 3z - 5 = 0$ and $3x - 2y - z + 1 = 0$ and cutting off equal intercepts on the x and z axes. (3× 2)

QUESTION-16

Find the value of λ for which the four points with position vectors $3\hat{\imath} - 2\hat{\jmath} - \hat{k}$, $2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$,

$-\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $4\hat{\imath} + 5\hat{\jmath} + \lambda\hat{k}$ are coplanar.

OR

Find the value of P for which the vectors $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$ and $\vec{b} = \hat{\imath} + P\hat{\jmath} + 3\hat{k}$ are i) perpendicular ii) parallel.

QUESTION-17

Find the area bounded by the parabola $y = x^2$ and $y = |x|$

OR

Find the area bounded by the curves $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$. (4)

QUESTION-18

Find the image of (1,2,-1) in the plane $2x + y - z = 2$. (6)

SECTION-C

QUESTION-19

(a) The marginal revenue function of a commodity is $9 + 2x - 6x^2$. Find the
 i) total revenue ii) demand function.

(b) the two lines of regression are $4x + 10y = 9, 6x + 3y = 4$. Find the line of regression of y on x.

(c) The average cost function for a commodity is given by $AC = x + 5 + \frac{36}{x}, x \neq 0$ in terms of output x. Find the total cost and marginal cost function .Also find the marginal cost when $x = 10$. (3× 2)

QUESTION-20

Fit a straight line to the following data treating 'y' as the dependent variable.

QUESTION-11

Show that a cylinder of a given volume which is open at the top , has minimum total surface area, provided its height is equal to the radius of its base. (6)

QUESTION-12

Evaluate

$$\int \frac{5x-2}{1+2x+3x^2}dx$$

OR

Evaluate

$\int_1^4 (x^2 - x)\, dx$ as the limit of a sum. (6)

QUESTION-13

A manufacturer has three machine operators A, B and C. The first operator A produces 1% of defective items, where as the other operators B and C produces 5% and 7 % defective items respectively. A is on job for 50% of the time, B on the job 30% of the time and C on the job for 20% of the time. All the items are put into one stockpile and then one item is chosen at random from this and found to be defective. What is the probability that it was produced by A? (6)

QUESTION-14

If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the following system of linear equations $x-2y = 10, 2x - y - z = 8, -2y + z = 7$

OR

By using elementary row transformations, find the inverse of $\begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ (6)

SECTION-B

QUESTION-15

(a) For any vector $\vec{a}$, prove that $|\vec{a} \times \hat{\imath}|^2 + |\vec{a} \times \hat{\jmath}|^2 + |\vec{a} \times \hat{k}|^2 = 2\,|\vec{a}|^2$.

(b) Find the angle between the following pairs of lines:

x	14	12	13	14	12
y	22	23	22	24	24

Predict the value of y when x = 16.

OR

If the regression equation of x on y is given by mx-y+10 = 0 and the equation of y on x is given by -2x+5y+14 =0. Find 'm' if the coefficient of correlation between x and y is $\frac{1}{\sqrt{10}}$. Also find the variance of y if the variance of x is 12. (4)

QUESTION-21

Given that the marginal cost and the average cost of a product are directly proportional to each other. Find the total cost function so that the cost of producing 2 units is ₹ 8 and that of producing 4 units is ₹ 64.

OR

The cost function of a firm is given by the equation $C = 300x-10x^2+\frac{x^3}{3}$ where 'x' stands for the output. Calculate the output at which

i) marginal cost is minimum

ii)average cost is equal to marginal cost (4)

QUESTION-22

A company manufactures three kinds of calculators A,B and C in its two factories I and II . The company has got an order for manufacturing at least 6400 calculators of kind A, 4000 of kind B and 4800 of kind C. The daily output of factory I is of 50 calculators of kind A , 50 calculators of kind B and 30 calculators of kind C. The daily output of a factory II is of 40 calculators of kind A, 20 of kind B and 40 of kind C . the cost per day to run factory I is ₹12000 and of factory II is ₹ 15000. How many days do the factories have to be in operation to produce the order with the minimum cost? Formulate this problem as an LPP and solve it graphically. (6)